MCG 1100 - Introduction to Mechanical Engineering

Dissection Lab 1(b) - Review of Free-Body Diagrams

(prepared by W. Hallett)

A free-body diagram (FBD - in French, schéma de corps isolé) is a sketch of a part or system isolated from its supports and other parts connected to it, showing all external forces acting on it. You should already know how to draw FBD's from GNG1105; this section is merely a summary. In drawing a FBD, think of drawing a box or a boundary surface around the object which separates it from other components; wherever this surface passes through a connection or support, replace the connection with appropriate forces.

7 Rules for drawing FBD's

- a FBD shows only **external** forces acting on the object. Any forces **inside** the FB are not "seen" on the diagram - you only "see" forces that are "cut" by the bounding surface. Think of the FB as a "black box", whose contents are invisible.

- a FBD shows the forces applied **to** the body **by** the supports and connections, **not** the other way around.

- in separating machines, structures, etc. into parts, the FBD's of individual parts must obey Newton's **third law of action and reaction**: at each connection, the directions of force arrows on connecting parts must be equal and opposite. The representation of forces must also be the same: do not show the force as a vector on one part and as components on the other.

- identify all **two-force members** - this will speed up solving for the forces. A two-force member has forces applied at only two points, has negligible weight, and has no couples (moments) applied to it. Forces on two-force members must be shown as vectors, not components, to indicate that the directions of these forces is known.

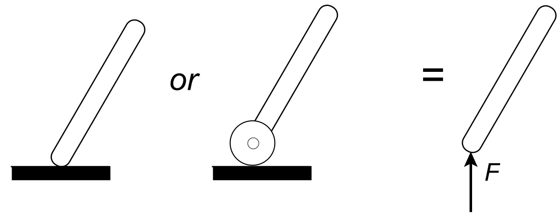
- all forces **must be labelled** with names that identify the variables to be used in calculations. Names are usually letters associated with points on the diagram, such as $\vec{A}$ for a force vector acting at point A or A_X, A_Y for its components. Two-force members are often labelled with the two points connected by the member: *e.g.* F_{BA} acts on point B in the direction of A. If forces are not labelled, the FBD is incorrect.

A FBD can be a very rough sketch, showing only essential features - an elaborate drawing is not necessary.

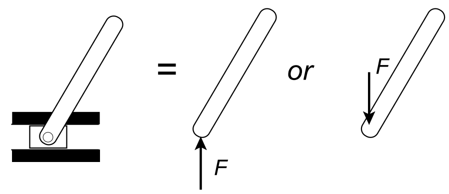
Standard Connections

In drawing free-body diagrams, look for standard supports and connections. Your textbook for GNG1105 (Beer and Johnston, *Statics*) has tables summarizing these in two- and three dimensions. Briefly, these standard connections are, in two dimensions:

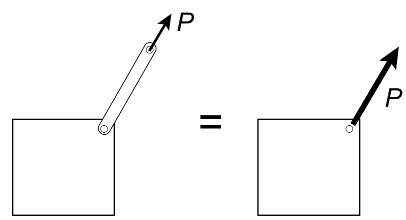
- frictionless contact, wheel or roller - force is always normal to the contact surface. Known force direction, unknown magnitude.



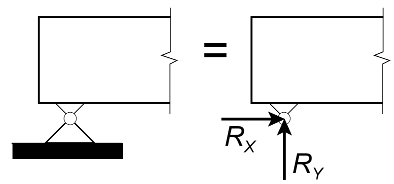
- frictionless slide - force is normal to contact, can be up or down depending on direction of other applied forces. Known force direction (but unknown sign), unknown magnitude.



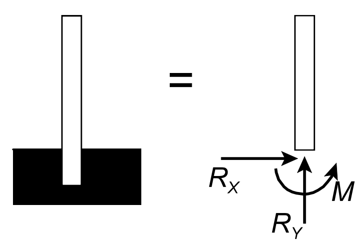
- the two-force member (usually a cable, chain or a light pin-jointed link) - force is in line with the member. Known force direction, unknown magnitude.



- the frictionless pin joint - has two unknown force components (equivalent to unknown direction and magnitude). **Exception**: if it connects to a two-force member, it has a vector with known force direction acting on it, the direction being that of the two-force member.

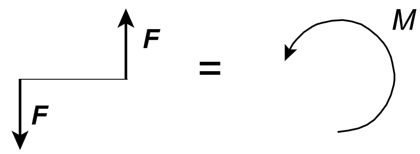


- the fixed or "built-in" support (moment-resisting connection) - two unknown force components and a couple (moment). An example is a post whose end is buried in the ground or in concrete, or a bar inserted into a hole.

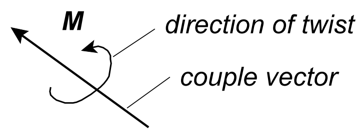


These are used as idealized models of real connections. Nothing in reality is completely frictionless, but for many sliding or rolling contacts as well as for bearings and pins the friction forces are much smaller than the other forces applied, so they may be neglected. Real belts, chains and cables are not completely flexible or weightless, but in most cases their weights and stiffnesses are small compared to the other forces, so they may be approximated as weightless and are therefore two-force members.

Moments, Couples and Torque



Couple in 2 dimensions



Couple in 3 dimensions

The terms "moment", "couple" and "torque" are often used interchangeably, although there are small differences in meaning between them. A couple is defined formally as two equal and opposite forces acting some distance apart. The two forces cancel, so the only effect of a couple is to apply a pure twisting action - a moment - with zero resultant force. The word "couple" is therefore used to refer to a pure moment regardless of how it is actually applied - it does not have to be associated with an actual pair of forces. A "torque" is a couple applied as the twisting action of a rotating shaft; "torque" and "couple" are essentially the same. (Another word sometimes used for this is "torsion".) The term "moment" is more general: it can refer to the moment of a force (the twisting action produced by a single force about a point at some distance from its line of action) or to the moment of a couple.

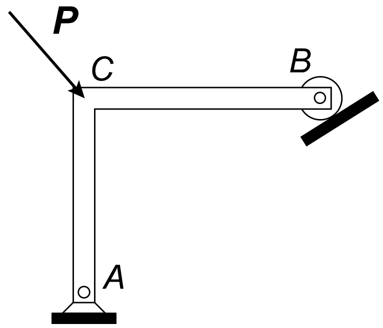
In two dimensions, a couple is shown on an FBD as a curved arrow. In three dimensions, it is shown as a curved arrow encircling a straight arrow; the latter represents the direction of the couple vector, which is parallel to the axis of the twisting action. The directions of the couple vector and the curved arrow are given by the right hand rule which defines the direction of the vector cross product (the mathematical definition of a moment).

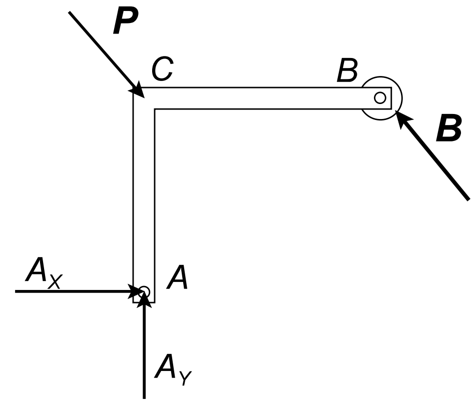
On a free-body diagram, the only moments which are shown are couples. A free-body diagram does **not** show the moments produced by individual forces acting on the system, as these moments and other effects of the forces are automatically included by placing the forces on the diagram.

8 Detailed Notes on Free-Body Diagram Practice

8.1 Direction/Sign of Unknown Forces

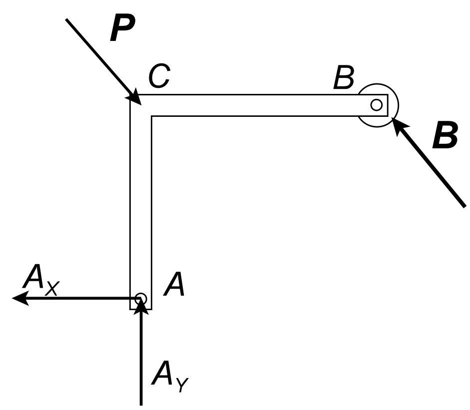
In drawing unknown forces on an FBD, the angle of the force may be known (*e.g.* a cable or thin link, for which the line of action is along the cable, or a frictionless slide or support, for which the force is normal to the contact surface). If it is not known (*e.g.* pin joint, fixed support), two unknown components should be specified. In both cases, one has to decide which direction or sign to give the force arrows on the diagram. Most people draw FBD's with unknown forces acting in the positive co-ordinate directions. If this turns out to be wrong for a particular force, the algebra will tell you by giving you a negative sign for the force.



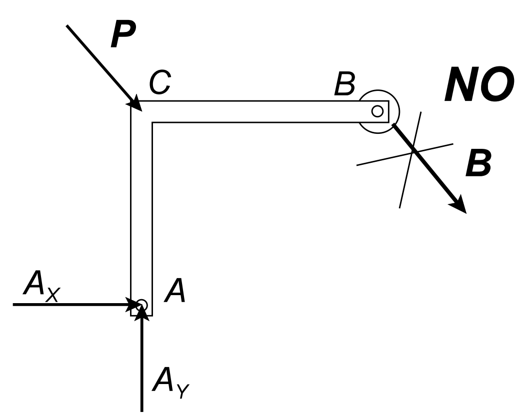


In the example above, force **A** has been entered as components in the positive x and y directions. Calculations (ΣM_C) would give a negative sign for A_X, showing that A_X in reality points in the negative x direction. If this happens, **DO NOT** go back and change the free-body diagram. Simply note in your solution that “positive force directions are as shown in the FBD”, or show the force direction unambiguously in giving your answer.

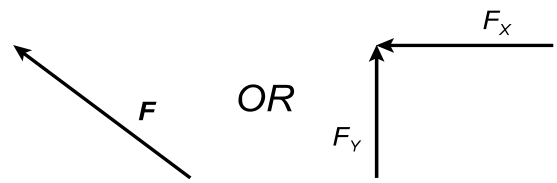
As an alternative to drawing all forces in positive co-ordinate directions, you can try to guess the force directions by mentally taking ΣM and ΣF as you draw the diagram. In this example, we know the directions of **P** (given) and **B** (frictionless support); mentally taking ΣM_C shows that A_X points in the negative x-direction. You will not always be able to get all force directions this way - for example, you cannot get the correct direction for A_Y by ΣM or ΣF_Y without doing actual calculations - but the exercise will help you understand how forces work, and will also help you plan your solution strategy. Both this FBD and the previous one are correct.



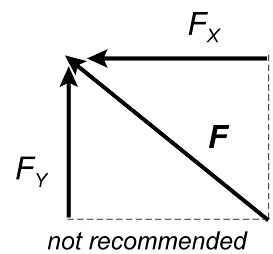
Exception: if the force direction is known (*e.g.* a cable is always in tension, the force at a support always acts to keep the surfaces in contact), the FBD **must** show the correct direction. Failure to do this will result in incorrect answers. The FBD at the right is wrong, because **B** must support the roller - as drawn, it is pulling the roller down instead. A solution based on this FBD will give **A** with the wrong direction.

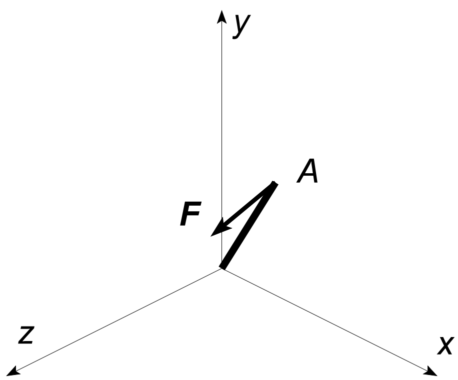


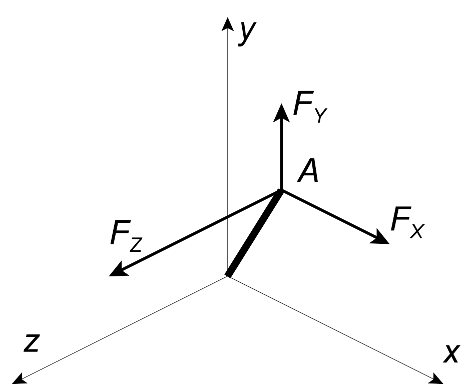
8.2 Components



Forces may be drawn as components if they make the problem clearer. (For a two-force member, you should *not* show components - see §8.4 below.) It is not usually good practice to show both the vector *and* its components, since the FBD then appears to show *three* unknowns instead of one or two. If however you do show both, use the same notation for both and connect them by dashed lines to make it clear that the components are not other forces (diagram at right). However, in two dimensions one will normally show only the components or only the vector, not both. In three dimensions, components are often easier to draw and interpret. The two forces below are equivalent, but the diagram showing components instead of the force vector is easier to read.

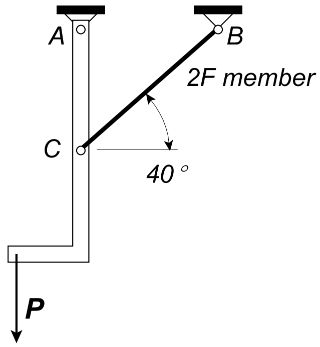


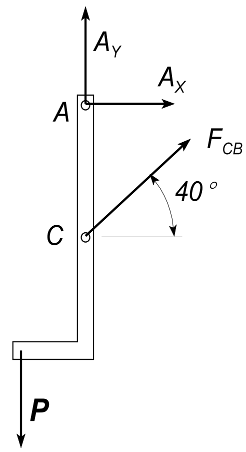


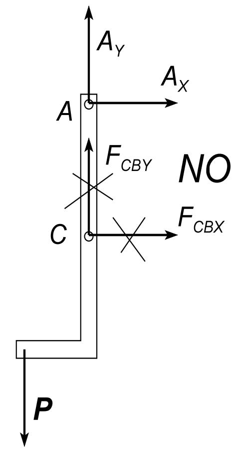


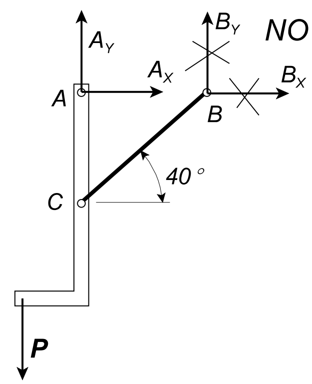
8.3 Forces must be labelled. Labelling is necessary to define your notation, so that the marker can understand the algebraic part of your solution. A FBD with the forces unlabelled is WRONG.

8.4 Two-force members. Identify **all** two-force members before drawing the FBD. The essential fact of a two-force member is that the forces on it are aligned with the two points of application. Forces exerted by two-force members must therefore **always** be shown as a vector lined up with the member - *do not show two-force member forces as components!* In the example below, the first FBD allows one to recognize immediately that the direction of $\mathbf{F_{CB}}$ is known, so that the only unknown is the magnitude F_{CB}. If components are shown instead (right hand sketch), it gives the impression that there are *two* unknowns - F_{CBX} *and* F_{CBY} - instead of only one. This is a common mistake, and can result in one not being able to solve a problem because it appears to have too many unknowns. If one draws the right hand FBD above, one has to recognise that $F_{CBX} = F_{CB} \cos 40°$, $F_{CBY} = F_{CB} \sin 40°$ to reduce the number of unknowns from 4 to 3.









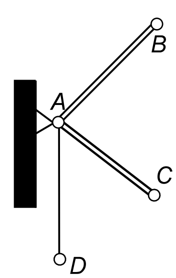
The mistake usually arises because most two-force members have pin joints, and pin joints are often drawn with two unknown force components. Make a habit, therefore, of checking each pin joint to see if it is attached to a two-force member. Another variation of this mistake is shown at right. It is not wrong to include member CB in the FBD, but the reaction at C must recognize that CB is two-force.

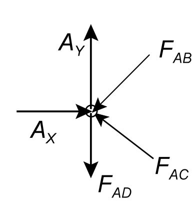
To identify two-force members, look for members that have the following characteristics:

- forces applied at only two points (usually the ends);
- no moments applied anywhere;
- negligible weight (weight would be a third force).

Most two-force members are cables, thin bars or links with pin joints. However, they can also have more complex shapes - *e.g.* curved bars or plates - and forces may be applied through point contacts or slides as well as through pin joints. A two-force member will *never* have a fixed (built-in) connection, because that would include a moment.

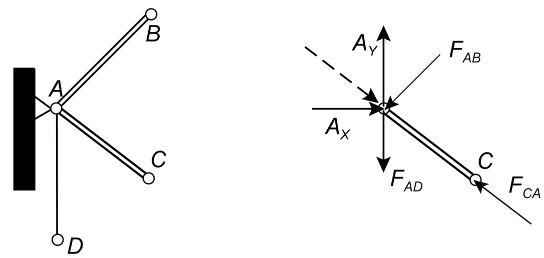
Often two or more two-force members meet at a pin joint. This occurs at every joint in a truss, but also in other structures and mechanisms. There are two ways to deal with this:





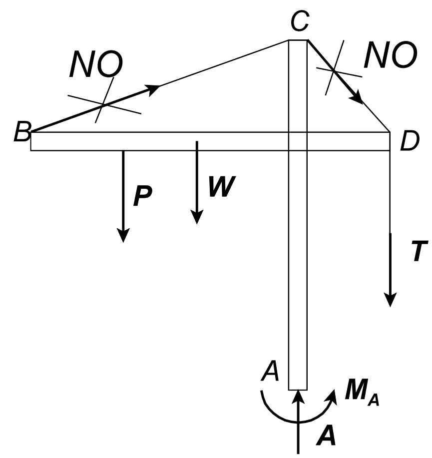
(a) take the pin as a separate free-body. This is what is done in method of joints analysis of a truss. The sketch shows two-force members joined at a pin support. A free-body of the pin shows the reaction forces A_X and A_Y at the support as well as the separate member forces F_{AB}, F_{AC} and F_{AD} all acting on the pin.

(b) take the pin and one of the members (AC in this example) as the FB, and draw the other forces acting on this FB (sketch at right). Note that the sum of forces acting at point A is equal to a resultant force (shown by the dashed line) that is in line with AC and equal and opposite to F_{CA}. *This resultant is included here for illustration purposes only and must not be shown on the FBD.*

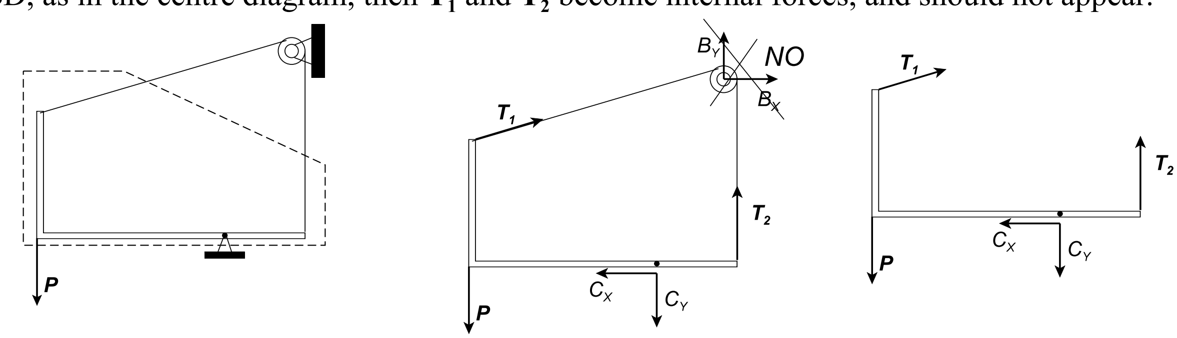


The second procedure is the one generally used if one or more of the members is multi-force (*i.e.* with forces applied at more than two points): the pin is assumed attached to the multi-force member, and all the other forces are drawn acting on it. Note that forces on a multi-force member do *not* line up with the member!

8.5 External Forces Only. A free-body diagram shows only forces acting external to the chosen free-body; internal forces must *never* be shown. The diagram at right is incorrect for this reason. All forces acting within an FB are in equilibrium, and do not affect the forces acting at the supports and external connections of the FB. At right, for example, the tensions in members BC and CD will not affect the reactions at all. Including these tensions in the solution would add non-existent forces to the object and make the solution incorrect.



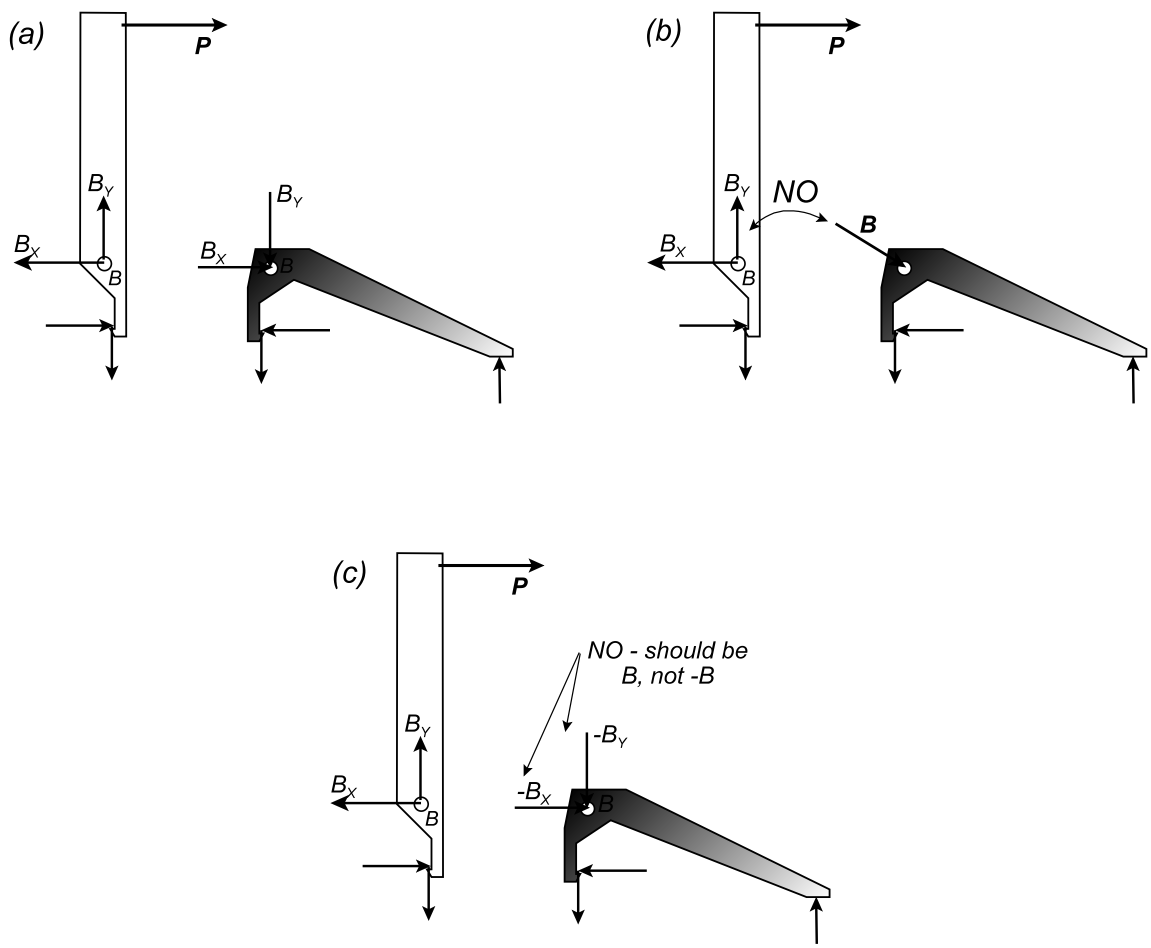
A good way to distinguish between internal and external forces is to draw a boundary around the FB: any forces or connections **intersected** by the boundary will appear on the FBD, anything **within** the boundary will not. In the centre diagram below, the lack of a clear definition for the FB has resulted in an improper FBD. One should draw a boundary as shown, excluding the pulley and resulting in the correct FBD at the right. Alternatively, if one wishes to include the pulley in the FBD, as in the centre diagram, then $\mathbf{T_1}$ and $\mathbf{T_2}$ become internal forces, and should not appear.



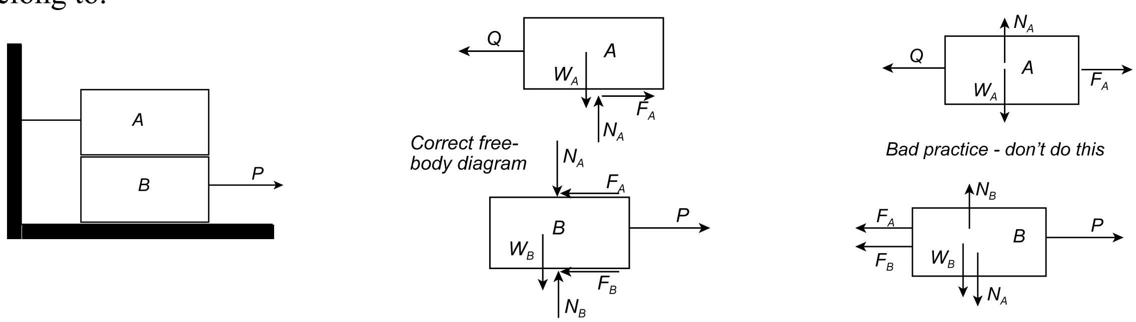
8.6 Newton's Third Law. In problems involving several components (*e.g.* frames and machines), in which FBD's for individual parts must be drawn, make sure that Newton's third law (action-reaction) is applied at each joint which is taken apart: force components acting on one member must be shown as equal and opposite on the other member that they connect to. The same is true of couples, for example at a fixed support. Examples are shown in the diagram at left below and in the diagram for item 7 further below. Two points to note:

(a) Draw the force in the same way on both parts. Don't draw the force as a single vector on one part and the same force as x and y components on the other - this is confusing (see (a) and (b) below).

(b) Do NOT put a negative sign on the force when you reverse the direction of the arrow (see (c) below). The Beer and Johnston textbook does this in a few places (*e.g.* Figures 6.20 - 6.23), but this practice is WRONG and will cause great confusion in setting up the equations. Note that the textbook is inconsistent here, because these incorrect figures are followed by examples which do *not* use this bad practice.



8.7 Locations of Forces. Forces should always be shown applied to the **points at which they act**. The centre sketch below shows the correct placement of forces for the friction problem at the left, with each contact surface having a frictional force and a normal force applied at the appropriate surface. The right hand sketch shows the same free-body diagram as drawn by some school physics textbooks: this is bad practice, because it is almost impossible to see which surfaces the forces belong to.



8.8 Weights. It is usual to neglect the weights of parts unless they are expressly given, and they will therefore not usually appear in the FBD. The weights of machine parts and structures are generally small compared with the loads they have to carry. Exceptions are large civil engineering structures, whose weights comprise a major part of the total load on the structure.

(last revision February 2016)